

Instanton induced charged fermion and neutrino masses in a minimal Standard Model scenario from intersecting D-branes

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Abstract

String instanton Yukawa corrections from Euclidean D-branes are investigated in an effective Standard Model theory obtained from the minimal $U(3) \times U(2) \times U(1)$ D-brane configuration. In the case of the minimal chiral and Higgs spectrum, it is found that superpotential contributions are induced by string instantons for the perturbatively forbidden entries of the up and down quark mass matrices. Analogous non-perturbative effects generate heavy Majorana neutrino masses and a Dirac neutrino texture with factorizable Yukawa couplings. For this latter case, a specific example is worked out where it is shown how this texture can reconcile the neutrino data.

1. Introduction

The recent years D-brane configurations with $U(3) \times U(2) \times U(1)^n$ gauge symmetry have been systematically investigated and extensively analyzed [1]-[11], particularly in the context of intersecting D-branes. This gauge symmetry could be considered as the natural successor of the Standard Model (SM) gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ in the context of D-branes. The constant endeavors to examine the low energy implications of the emerging effective low energy theory during the last few years have been absolutely justified, as many of these efforts were crowned with considerable progress towards a successful D-brane realization of the SM gauge group with its chiral and Higgs matter content.

In the intersecting D-brane constructions the SM chiral matter arises in the intersection of the brane-stacks wrapping three cycles in the internal six-dimensional manifold. Wrapping branes on a compact manifold intersect each other, while the number of fermion fields equals the number of intersections, a fact that eventually reveals a possible geometrical origin of the fermion family multiplicity. The emerging effective field theory model usually consists of the SM gauge symmetry augmented by some global $U(1)$ symmetries, with the minimal fermion and Higgs spectrum escorted (in several cases) by some additional -hopefully innocuous- matter representations.

One of the main shortcomings of these constructions is the absence of several Yukawa couplings necessary to provide all quark and lepton fields with non-zero masses. Indeed, a generic feature in D-brane constructions is that ordinary matter fields are charged under the aforementioned global $U(1)$ symmetries accompanying the SM gauge group. Undoubtedly, these symmetries impose additional restrictions (beyond those of the SM gauge symmetry) on the perturbative Yukawa superpotential terms and as a consequence, some of the SM fermion fields remain massless.

It was shown that these global $U(1)$'s are broken from stringy instantons effects [12, 13, 14, 15] and as a consequence new, non-perturbative contributions can be generated to the superpotential. In type IIA compactifications in particular, the eventual candidates are the Euclidean D2 instantons (subsequently called $\mathcal{E}2$ instantons for short) wrapping three cycles in the internal manifold. If they are chosen to intersect with the relevant D6-branes appropriate number of times, there exist inextricable superpotential terms filling in the zeros of the fermion mass matrices. Compared to the perturbative Yukawa couplings, the instanton induced terms appear to be suppressed by the exponential of the classical instanton action $e^{-S_{\mathcal{E}2}}$. These corrections although subdominant with respect to the tree-level perturbative Yukawa couplings, might prove of vital importance as far as the viability of the construction is concerned. Indeed, in the class of models under consideration, these instanton induced terms could offer a natural explanation for the observed hierarchy of the fermion mass spectrum, acting as a surrogate for missing non-renormalizable terms of the perturbative sector.

In the present work we will examine the stringy instanton effects in the case of a minimal D-brane set-up required to embed the SM gauge symmetry. This configuration consists of three brane-stacks leading to $U(3) \times U(2) \times U(1)$ gauge symmetry with the ordinary quarks and lepton fields living in their intersections. It was shown that three different

fermion embeddings are possible[6], while, due to the minimal structure of this configuration some particular cases can be naturally embedded in higher unified symmetries such as the $U(5)$ [16], the trinification [17] or the Pati-Salam[18] D-brane analogue[19].

The paper is organized as follows. In the next section we briefly present the derivation of the model from the particular D-brane set-up and give a short description of its embedding in the intersecting D-brane scenario. We introduce the chiral and Higgs spectrum and emphasize the salient features of the model, including the unification prospects and the perturbative superpotential structure for several choices of the matter content. In section 3, we recapitulate the basic procedure on the derivation [12, 13] of the stringy instanton non-perturbative Yukawa couplings and apply the results to the specific D-brane construction analyzed in this work. We derive the non-perturbative contributions for the quark and neutrino mass matrices and comment on the peculiar nature of the resulting Yukawa textures. A separate discussion is devoted to the attractive Dirac neutrino mass matrix texture in section 4 and emphasis is given in a specific example which reconciles the experimental data. Finally, in section 5 we present our conclusions.

2. The Standard Model on a $U(3) \times U(2) \times U(1)$ D-brane set-up

We start this section with a brief description of the simplest model constructed by the minimum possible number of brane-stacks. The SM embedding in a D-brane configuration has received much attention the last decade and attempts to reproduce its spectrum and low energy successful reconciliation with all the known experimental data has stimulated a thorough investigation of the configurations with several sets of brane-stacks leading to the enhanced SM symmetry

$$U(3) \times U(2) \times U(1)^n \rightarrow SU(3) \times SU(2) \times U(1)^{n+2} \supset G_{SM} \quad (1)$$

The integer power n represents the number of $U(1)$ branes needed to reproduce the desired SM spectrum while a linear combination of the $n + 2$ abelian factors will accommodate the hypercharge generator.¹ SM matter fields are ‘incarnated’ through open strings with their endpoints on the intersecting D6-branes and are localized in the intersection locus of the latter. In type IIA orientifolds the following types of representations arise:

Open strings appearing in the intersection locus of two $D6_a, D6_b$ branes (ab for short) ‘create’ bifundamental representations (N_a, \bar{N}_b) while their multiplicity is given by the number of intersections

$$I_{ab} = \prod_{i=1}^3 (m_{ai}n_{bi} - m_{bi}n_{ai}) \quad (2)$$

¹ From (1) we can observe that due to the group relation $U(N) \rightarrow SU(N) \times U(1)$, one might assume that the SM gauge symmetry could be generated with only the two brane stacks $U(3), U(2)$, so we could take $n = 0$. It can be easily shown that the hypercharge generator cannot be expressed only in terms of these two $U(1)$ symmetries, so we need $n \geq 1$.

The six-dimensional internal manifold is assumed to be factorizable with a torus geometry, $T^6 = T_1^2 \times T_2^2 \times T_3^2$ while here the (n_{ai}, m_{ai}) pairs represent the winding numbers of the $D6_a$ -stack around the two radii of the i^{th} torus.

Chiral fermions appear also in (N_a, N_b) representations from the intersections of a $D6_a$ brane-stack with a mirror brane $\Omega\mathcal{R}D6_b \rightarrow D6_{b^*}$. Their number is given by

$$I_{ab^*} = - \prod_{i=1}^3 (m_{ai}n_{bi} + m_{bi}n_{ai}) \quad (3)$$

States arising in the intersection of a $D6_a$ -brane with its corresponding mirror one, may belong to the antisymmetric or symmetric representations. Those which remain invariant under the combined $\Omega\mathcal{R}$ action belong to the antisymmetric representations of the $U(N_a)$ gauge group and their multiplicity (denoted as $I_{aa^*}^A$) is given by

$$I_{aa^*}^A = 8m_{a1}m_{a2}m_{a3} \quad (4)$$

Depending on the specific winding numbers, there may also appear symmetric and anti-symmetric representations with equal multiplicities

$$I_{aa^*}^{A,S} = 4m_{a1}m_{a2}m_{a3} (n_{a1}n_{a2}n_{a3} - 1) \quad (5)$$

Finally, additional restrictions are imposed on the (n_{ai}, m_{ai}) sets originating from the RR -type tadpole conditions which read:

$$T_0 = \sum_a N_a n_{a1}n_{a2}n_{a3} = 16 \quad , \quad T_i = \sum_a N_a n_{ai}m_{aj}m_{ak} = 0, \quad i \neq j \neq k \neq i \quad (6)$$

where the indices i, j, k take the values 1, 2, 3 and refer to the three torii T_j^2 .

Returning to (1), we have already asserted that the three SM gauge factors need at least three brane-stacks, however, if one insists to accommodate the SM fermions only in bifundamentals, then one should take at least $n = 2$ [11, 8]. If, for the sake of simplicity and ‘economy’ one uses the possible minimal brane set-up, i.e. when $n = 1$, some of the fermions are accommodated in bifundamentals obtained in the intersections of two different brane-stacks, while others are accommodated in antisymmetric or symmetric representations which can be created by strings with endpoints on a given brane-stack and its corresponding mirror. As mentioned above, there are three distinct arrangements of the SM-fields (see [6], [8]) and here we concentrate on the configuration depicted in figure 1. For later convenience we call a, b, c the stacks related to $U(3), U(2), U(1)$, respectively. We start exploring the model implementing the minimal fermion and Higgs spectrum of our particular D-brane set-up which is shown in Table 1. This consists of the three SM fermion generations, their corresponding right handed neutrinos and one pair of Higgs doublets. The spectrum is consistent with the following hypercharge definition

$$Y = \frac{1}{6} \mathcal{Q}_a - \frac{1}{2} \mathcal{Q}_c \quad (7)$$

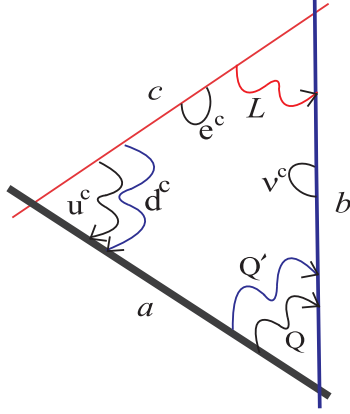


Figure 1: A depiction of the $U(3) \times U(2) \times U(1)$ D-brane configuration with strings representing the SM states. For the sake of simplicity, a, b, c denote $U(3), U(2)$ and $U(1)$ branes respectively. In this figure D-branes are not distinguished from their corresponding mirrors. Thus, the blue string representing the quark doublet Q' is stretched between the $D6_a$ and the mirror $D6_{b^*}$. Similarly, one endpoint of the “ d^c -string” is attached on the mirror $D6_{c^*}$.

Inters.	$SU(3) \times SU(2)$	\mathcal{Q}_a	\mathcal{Q}_b	\mathcal{Q}_c	Y
ab	$1 \times Q(3, \bar{2})$	1	-1	0	$\frac{1}{6}$
ab^*	$2 \times Q'(3, 2)$	1	1	0	$\frac{1}{6}$
ac	$3 \times u^c(\bar{3}, 1)$	-1	0	1	$-\frac{2}{3}$
ac^*	$3 \times d^c(\bar{3}, 1)$	-1	0	-1	$\frac{1}{3}$
bc	$3 \times L(1, \bar{2})$	0	-1	1	$-\frac{1}{2}$
cc^*	$3 \times e^c(1, 1)$	0	0	-2	1
bb^*	$3 \times \nu^c(1, 1)$	0	-2	0	0
bc^*	$1 \times H_d(1, 2)$	0	1	1	$-\frac{1}{2}$
	$1 \times H_u(1, \bar{2})$	0	-1	-1	$\frac{1}{2}$

Table 1: The quantum numbers of the SM fermions in the $U(3) \times U(2) \times U(1)$ brane configuration. The last column is the Hypercharge $Y = \frac{1}{6}\mathcal{Q}_a - \frac{1}{2}\mathcal{Q}_c$ while three previous ones refer to the $U(1)$ charges with respect to the a, b, c brane-stacks.

where $\mathcal{Q}_{a,c}$ are the $U(1)_{a,c}$ charges of the SM representations while the corresponding $U(1)_Y$ gauge coupling a_Y is the combination

$$\frac{1}{a_Y} = \frac{1}{6} \frac{1}{a_3} + \frac{1}{2} \frac{1}{a_1} \quad (8)$$

It turns out that the $U(1)$ -normalization coefficient is given by $k_Y = \frac{1}{6}\xi + \frac{1}{2}\xi'$ where $\xi = \frac{\alpha_2}{\alpha_3}, \xi' = \frac{\alpha_2}{\alpha_1}$ are the gauge coupling ratios from which we deduce the Weinberg angle at the String scale M_S ,

$$\sin^2 \theta_W = \frac{1}{1 + k_Y} = \frac{6}{6 + \xi + 3\xi'}$$

In the particular case of a unified value of all three gauge couplings a_1, a_2, a_3 , the string scale M_S turns out to be rather high. For partial unification $a_2 = a_3$ the ‘standard’ GUT value $\sin^2 \theta_W = \frac{3}{8}$ could be obtained for $\xi' = 3$, whilst for the standard MSSM spectrum and a SUSY scale $m_s \sim m_Z$, non-abelian gauge unification ($a_2 = a_3$) implies

$$M_S = e^{\frac{\pi}{2} \left(\frac{\sin^2 \theta_W}{\alpha} - \frac{1}{\alpha_3} \right)} \left(\frac{m_Z}{m_s} \right)^{\frac{13}{12}} m_Z \sim \mathcal{O}(10^{16}) \text{GeV} \quad (9)$$

In D-brane scenarios each gauge group factor is related to a brane-stack which occupies certain dimensions of the higher dimensional space. Since the present model is based on a product of gauge groups, these correspond to an equivalent number of brane-stacks which in general span different directions of the 10-d space. Thus, gauge couplings in general have different values and the String scale may be significantly lower in intersecting scenarios. A detailed analysis of the unification prospects and many other low energy phenomenological issues in the context of intersecting branes has been presented elsewhere [9].

In the present setup, one quark doublet (namely $Q = (3, \bar{2})$), the right-handed up-quarks u^c , and the lepton doublets \mathcal{L} arise from the intersection of $D6_a - D6_b$, $D6_a - D6_c$ and $D6_b - D6_c$ branes respectively. In this case, their number is equal to the corresponding intersections I_{ab} , I_{bc} , I_{ac} , determined by the formula (2).

Two quark doublets belong to the $(3, 2)$ representation of the $U(3) \times U(2)$ brane-stacks thus they arise from the intersections of $D6_a$ with the mirror D-brane $\Omega \mathcal{R} D6_b \rightarrow D6_{b^*}$, hence their number is determined by the formula (3). The remaining chiral states arise in the intersection of a $D6_x$ -brane ($x = b, c$) with its corresponding mirror one, while they could belong to the antisymmetric or symmetric representations, their multiplicity denoted $I_{xx^*}^A$ and $I_{xx^*}^S$ respectively and given by (4) and (5) analogously. In particular, right-handed electrons arise in the cc^* intersection and they belong to the antisymmetric representation, their $U(1)_c$ charge is $\mathcal{Q}_c = -2$, its absolute value being twice as big compared to that of a bifundamental. In the bb^* intersection on the other hand, since the $U(1)_b$ generator does not participate in the hypercharge, the corresponding antisymmetric representations are electrically neutral and can be identified with the right handed neutrinos, while their $U(1)_b$ -charge can have any of the two possible values $\mathcal{Q}_b = \pm 2$.

As can be inferred from the above discussion, we have chosen in this minimal approach to derive the quark doublets from two different intersections. Thus, in Table 1 we observe that one left-handed quark doublet is in fact an $SU(2)$ -antidoublet since it arises from the ab sector, while the remaining two quark families are $SU(2)$ -doublets, since they come from the ab^* sector. The reason for this arrangement [3] is that the $SU(N)$ cancelation conditions demand the same number of N and \bar{N} ’s for any $U(N)$ gauge group factor and this has to be satisfied even for the $SU(2)$ gauge group, hence one has to distinguish between doublets and anti-doublets so we can ensure that they appear in equal numbers. Thus, a consistent derivation of the minimum SM spectrum of the model requires $I_{ab} = 1, I_{ab^*} = 2$ for the left-handed quarks, $I_{ac} = I_{ac^*} = -3$ for up and down right handed quarks, and $I_{bc} = -3$ for the lepton doublets. Furthermore, the conditions $I_{cc^*} = I_{bb^*} = -3$ ensure the existence of three right-handed electrons e^c and neutrinos ν^c and finally $I_{bc^*}^{N=2} = 1$ gives one MSSM

Higgs pair². The wrapping numbers (n_{ai}, m_{ai}) are also subject to additional restrictions from Tadpole cancelation conditions (6) which usually impose restrictions on extra hidden matter related to strings attached to bulk branes.

The tree-level quark and lepton Yukawa couplings of this minimal chiral and Higgs spectrum which is consistent with the above hypercharge assignment are

$$\mathcal{W} \supset \lambda_{pj}^u Q'_p u_j^c H_u + (\lambda_j^d Q d_j^c + \lambda_{ij}^l L_i e_j^c) H_d \quad (10)$$

Let us state a few remarks for the resulting superpotential, starting from the quark sector. In the first term, the indices i, j run over all three fermion generations, while p takes only two values. Thus, only two doublet quark flavors contribute through tree-level perturbative Yukawa couplings in the up-quark mass matrix. The reason is that the additional $U(1)_{(a,b,c)}$ charges carried by the various representations do not allow for a coupling involving the representation $Q(3, 2)_{(1, -1, 0)}$ generated in the intersection of the ab brane-stacks. For the same reason tree-level mass terms for the two quark doublets living in the intersection ac^* do not appear in the down quark mass matrix, since the down right-handed quarks couple solely to the remaining quark doublet.

Depending on the particular assignment of the fermion generation, three possible mass matrix textures arise. Thus, if the two Q'_p quark doublets correspond to the first two generations, then we obtain the following up and down quark Yukawa textures

$$m_Q = \begin{pmatrix} \lambda_{11}^u & \lambda_{12}^u & \lambda_{13}^u \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ 0 & 0 & 0 \end{pmatrix} \langle H_u \rangle, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda_{31}^d & \lambda_{32}^d & \lambda_{33}^d \end{pmatrix} \langle H_d \rangle \quad (11)$$

Clearly, in the same way, we have the following two remaining distinct possibilities, namely

$$m_Q = \begin{pmatrix} \lambda_{11}^u & \lambda_{12}^u & \lambda_{13}^u \\ 0 & 0 & 0 \\ \lambda_{31}^u & \lambda_{32}^u & \lambda_{33}^u \end{pmatrix} \langle H_u \rangle, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{21}^d & \lambda_{22}^d & \lambda_{23}^d \\ 0 & 0 & 0 \end{pmatrix} \langle H_d \rangle \quad (12)$$

and

$$m_Q = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ \lambda_{31}^u & \lambda_{32}^u & \lambda_{33}^u \end{pmatrix} \langle H_u \rangle, \quad m_d = \begin{pmatrix} \lambda_{11}^d & \lambda_{12}^d & \lambda_{13}^d \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle H_d \rangle \quad (13)$$

As can be seen, there is a complementary texture zero structure of the up and down quark mass matrices at the perturbative level, in the sense that the zero entries of the first are non-zero in the second and vice versa. The correct choice of family assignment will be dictated when the order of magnitude of the instanton and/or possible other contributions will be specified. Thus, a conclusive answer to this question, depends on the particular mechanism which will be adopted to generate the missing terms.

²We should note that the last three conditions, and in particular $I_{cc^*} = I_{bb^*} = -3$ are not mandatory and can be replaced with the less tight conditions $|I_{cc^*}|, |I_{bb^*}| > 3$. The existence of an extra right-handed electron e'^c and a neutrino ν'^c , under certain circumstances could explain the reported high energy positron excess through a coupling $e^c e'^{c*} \nu'^c$ [20].

Inters.	$SU(3) \times SU(2)$	\mathcal{Q}_a	\mathcal{Q}_b	\mathcal{Q}_c	Y
bb^*	$\nu'^c(1, 1)$	0	+2	0	0
bc	$H'_d(1, 2)$	0	−1	1	$-\frac{1}{2}$
	$H'_u(1, \bar{2})$	0	1	−1	$\frac{1}{2}$

Table 2: The additional Higgs representations with their quantum numbers.

As far as the lepton sector is concerned, Yukawa terms exist for all charged leptons, but Dirac and Majorana mass terms for the neutrinos are absent, hence they remain massless at tree level.

There are several possible solutions to this issue, among them the most obvious are those involving suitable extensions of the Higgs sector³. The most exciting and economic alternative however, is that of invoking stringy instanton effects to fill in the gaps in the mass matrices and create other useful couplings.

We start the analysis with the first alternative. Our initial suggestion was confined to the D-brane realization of the SM model with exactly the minimal chiral content and only one pair of Higgs doublets. As a result, a number of useful Yukawa couplings were missing at the perturbative superpotential. The most obvious possibility of deriving the requested couplings is to extend the Higgs sector introducing new multiplets with suitable $U(1)$ charges. The additional fields with their corresponding quantum numbers are shown in Table 2. These include an additional right-handed singlet neutrino supermultiplet with $U(1)_b$ charge $\mathcal{Q}_b = +2$ and one more Higgs pair emerging in the intersection of branes $D6_b$ and $D6_c$. Taking into account the fields of Table 2, the following tree-level Yukawa terms should be included to the superpotential

$$\mathcal{W}' = \lambda_j^u Q u_j^c H'_u + \lambda_j^d Q'_p d_j^c H'_d + \lambda_{ij}^{\nu} \mathcal{L}_i \nu_j'^c H_u \quad (14)$$

These new terms are sufficient to provide the missing entries in the quark mass matrices and generate the Dirac-type neutrino masses.

It is worth mentioning that the introduction of the second neutrino supermultiplet allows for an alternative option. It can be seen that the following fourth order non-renormalizable terms are permitted by the SM gauge and the three global $U(1)$ symmetries

$$\frac{1}{M_S} Q u_j^c H_u \nu'^c + \frac{1}{M_S} Q'_p d_j^c H_d \nu^c \quad (15)$$

Assuming a vacuum expectation value to a combination $\tilde{\nu}_H$ of the scalar partners $(\tilde{\nu}^c, \tilde{\nu}'^c)$ we observe that both terms are contributing to the zero entries of the quark mass matrices. In this case one dispenses with the use of the second Higgs pair, since in the new couplings (15) only the first pair of Higgs fields participates. This second choice however, discriminates

³ Of course we may adopt a more radical approach and assume that the $SU(2)$ cancelation conditions are satisfied by an appropriate number of $SU(2)$ antidoublets living in the Hidden sector[21]. Then, all left-handed quarks could originate from the same sector in the intersection of the $D6_a, D6_b$ brane-stacks, so that tree-level couplings for all families are permitted.

between the tree-level entries (10) and the new contributions (15) in the mass matrices, since the latter are suppressed relative to the first by the factor $\frac{\langle \tilde{\nu}_H \rangle}{M_S}$ and considerable adjustment of the parameters is needed to derive the observed quark mass hierarchy.

Next, instead of expanding the chiral and Higgs spectrum, as a second alternative, we consider stringy instantons and take into account their induced non-perturbative contributions to the perturbative superpotential (10). As in the case of the non-renormalizable terms (15), these corrections are also expected to be suppressed relative to the tree-level entries, the suppression factor being now proportional to the exponential of the volume of the cycle wrapped by the instanton in the internal manifold. We analyze these issues in the next section.

3. String Instanton induced masses

We have already realized that the D-brane embedding of the SM implies additional restrictions on the Yukawa superpotential due to the presence of the three $U(1)_{a,b,c}$ abelian symmetries. As a consequence, a number of useful Yukawa couplings crucial for the fermion mass terms are absent at the tree level of the perturbative sector.

In general, if $\Phi_{a_j b_j}, j = 1, 2, \dots, J$ represent superfields generated in the intersection of $D6$ brane-stacks, it is possible that the coupling $\prod_j \Phi_{a_j b_j}$ although invariant with respect to the non-abelian part of the gauge symmetry, might violate some abelian global $U(1)_a$ factor.

It has been recently suggested [12, 13] that in viable models, the missing tree-level Yukawa couplings could be generated from non-perturbative effects. In particular, considering $\mathcal{E}2$ instantons in type IIA string theory having appropriate number of intersections with the $D6$ -branes, a non-perturbative term

$$\mathcal{W}_{n.p.} \supset \prod_{j=1}^J \Phi_{a_j b_j} e^{-S_{\mathcal{E}}} \quad (16)$$

is induced, where the instanton action $S_{\mathcal{E}}$ can absorb the $U(1)_a$ charge excess of the matter fields operator $\prod_j \Phi_{a_j b_j}$. The global $U(1)_a$ charges carried by the instanton action have their microscopic origin in the extra fermionic zero modes living in the intersection of the $\mathcal{E}2$ -brane with the $D6_a$ brane-stack. Under the aforementioned abelian symmetry the transformation property of the exponential instanton action is

$$e^{-S_{\mathcal{E}}} \rightarrow e^{-S_{\mathcal{E}}} e^{i \mathcal{Q}_a(\mathcal{E}2) \Lambda_a} \quad (17)$$

where $\mathcal{Q}_a(\mathcal{E}2)$ represents the amount of the $U(1)_a$ -charge violation by the $\mathcal{E}2$ instanton. For an instanton wrapping appropriate number of cycles, this can be made exactly opposite to the total charge of the operator $\prod_j \Phi_{a_j b_j}$, so the coupling (16) is also $U(1)_a$ -invariant. Besides, the induced coupling (16) involves an exponential suppression by the classical instanton action $\mathcal{W}_{n.p.} \propto \exp\left\{-\frac{8\pi^2 \text{Vol}_{\mathcal{E}}}{g_a^2 \text{Vol}_{D6_a}}\right\}$ which, as can be seen depends on the volume $\text{Vol}_{\mathcal{E}2}$ of the cycle wrapped by the instanton and is inversely proportional to the perturbative

string gauge coupling g_a^2 . It is expected that even in the case of small g_a , string instanton contributions might become relevant.

In computing these non-perturbative effects the important ingredients are the instanton fermionic zero modes. We distinguish two different classes of them with respect to the charge they carry under the abelian gauge factors. In one class there are two uncharged fermionic zero modes θ_i with both endpoints attached on the $\mathcal{E}2$ instanton and they correspond to the two broken supersymmetries. In a second class are classified all the charged fermionic zero modes which are of primary importance in the computation of the instanton effects. These appear in the intersection of the $\mathcal{E}2$ instanton with a certain $D6_a$ brane-stack, while their relevance to a specific superpotential coupling is related to the number of $\mathcal{E}2 - D6_a$ intersections. In particular, the crucial quantity which appears in the coupling (16) is the charge $\mathcal{Q}_a(\mathcal{E}2)$ which is related to the $\mathcal{E}2$ intersections with the $D6$ brane stacks as follows. The $\mathcal{E}2$ instanton brane is considered wrapping a homological three-cycle $\pi_{\mathcal{E}}$ in the internal manifold, while is localized in the four-dimensional space-time. If π_a, π_{a^*} are the homological three-cycles of the $D6_a$ brane-stack and its mirror respectively, then

$$\mathcal{Q}_a(\mathcal{E}2) = -\mathcal{N}_a \pi_{\mathcal{E}} \circ (\pi_a - \pi_{a^*}) \equiv -\mathcal{N}_a (I_{\mathcal{E}a} - I_{\mathcal{E}a^*}) \quad (18)$$

where the $I_{\mathcal{E}a}$ and $I_{\mathcal{E}a^*}$ stand for the relevant intersection numbers. In what follows, we will consider a class of rigid $O(1)$ instantons, wrapping a rigid orientifold-invariant cycle in the internal space, where due to the $\mathcal{E}2 - a$ and $\mathcal{E}2 - a^*$ identification the charge (18) simplifies to

$$\mathcal{Q}_a(\mathcal{E}2) = -\mathcal{N}_a \pi_{\mathcal{E}} \circ \pi_a \equiv -\mathcal{N}_a I_{\mathcal{E}a} \quad (19)$$

Assuming appropriate number of wrappings, it is possible to cancel exactly the $U(1)_a$ charge excess and make the desired coupling invariant.

Returning to the specific model discussed in this work, we can check from (10) that there are several missing superpotential couplings, including those for the up- and down quarks and the right-handed Majorana neutrino masses. The absence of these terms make the model rather unrealistic at the perturbative level. We will see that instanton contributions do play a vital role on the issues of viability and phenomenological consistency of this construction.

We start with the up quark coupling $Qu_j^c H_u$ which, as can be observed, violates the $U(1)_b$ charge by two units

$$\mathcal{Q}_{bQ} + \mathcal{Q}_{b_{u^c}} + \mathcal{Q}_{b_{H_u}} = -2$$

and therefore cannot exist at tree-level. Assuming a $\mathcal{E}2$ instanton intersecting with the $D6_b$ brane-stack, so that

$$\mathcal{Q}_b(\mathcal{E}2) = -\mathcal{N}_b I_{\mathcal{E}b} \quad (20)$$

we conclude that $I_{\mathcal{E}b} = -1$ eliminates the charge-excess and the relevant coupling is allowed. The intersections of $\mathcal{E}2$ with the relevant branes is depicted in fig 1. At the computational level, one has to integrate over all instanton zero modes, the path integral being

$$\int \{d^4x d^2\theta d^2\lambda_b\} e^{-S_{\mathcal{E}}} y_j^u \epsilon_{mn} \epsilon_{r\ell} < \lambda_b^m Q^n u_j^c H_u^r \lambda_{b^*}^s > e^{Z'} \quad (21)$$

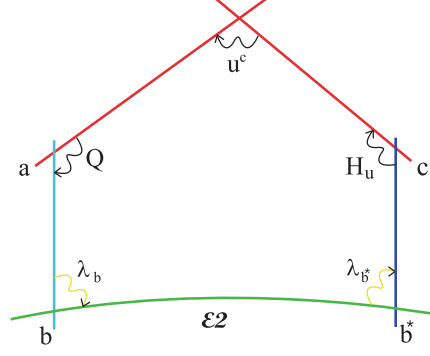


Figure 2: Stringy instanton contributions to the up-quark Yukawa mass matrix.

The integration (21) is over the four bosonic x^μ and two fermionic θ^i zero modes as well as the two instanton modes λ_b . Further there is an overall exponential suppression of the instanton action which makes the coupling invariant and $e^{Z'}$ in the regularized one-loop amplitude. Finally, m, n, r, s are $SU(2)$ indices, while y_j^u is a coupling constant with j running over all family indices. Clearly, the above instanton induced terms contribute to the zeros of the up-quark matrices presented in the previous section.

We turn now our attention to the down quark mass matrix. The trilinear couplings $Q'_p d_j^c H_d$ violate the $U(1)_b$ charge by two units $\mathcal{Q}_{b_{Q'}} + \mathcal{Q}_{b_{d^c}} + \mathcal{Q}_{b_{d_u}} = 2$, hence this term is also forbidden in the perturbative superpotential. In an analogous manner with the up-quark case discussed above, we introduce here a second instanton $\mathcal{E}2'$ which intersects with $D6_b$ and $D6_{b^*}$ brane-stacks, so that $\mathcal{Q}_b(\mathcal{E}2') = -\mathcal{N}_b I_{\mathcal{E}2'b}$, with $I_{\mathcal{E}2'b} = +1$. An integration over the instanton zero modes as in the case of (21) will lead to the effective instanton induced down quark Yukawa coupling $Q'_p d_j^c H_d$, suppressed by analogous exponential factors. Summarizing, the non-perturbative contributions to quark mass matrices induced by string instantons are

$$\mathcal{W}_{n.p.} = \lambda_j'^u Q u_j^c H_u + \lambda_{pj}'^d Q'_p d_j^c H_d \quad (22)$$

where the couplings $\lambda_j'^u$ and $\lambda_{pj}'^d$ are suppressed compared to the perturbative ones, due to the exponential factors involving the classical instanton action. Taking into account the non-perturbative contributions the mass matrices obtain the following texture form

$$m_Q = \begin{pmatrix} \lambda_{11}^u & \lambda_{12}^u & \lambda_{13}^u \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ \lambda_1'^u & \lambda_2'^u & \lambda_3'^u \end{pmatrix} \langle H_u \rangle, \quad m_d = \begin{pmatrix} \lambda_{11}'^d & \lambda_{12}'^d & \lambda_{13}'^d \\ \lambda_{21}'^d & \lambda_{22}'^d & \lambda_{23}'^d \\ \lambda_{31}^d & \lambda_{32}^d & \lambda_{33}^d \end{pmatrix} \langle H_d \rangle \quad (23)$$

and analogously for the other two cases (12,13). We should point out that in general, fermion mass textures as those above are rather unusual and considerable adjustment is needed to obtain the known up and down quark mass hierarchies. The rather striking fact however, is that since the non-perturbative couplings are suppressed by factors of the form e^{-S_ε} , relative to the perturbative ones, the quark mass ratios are directly correlated to the geometry of the internal manifold.

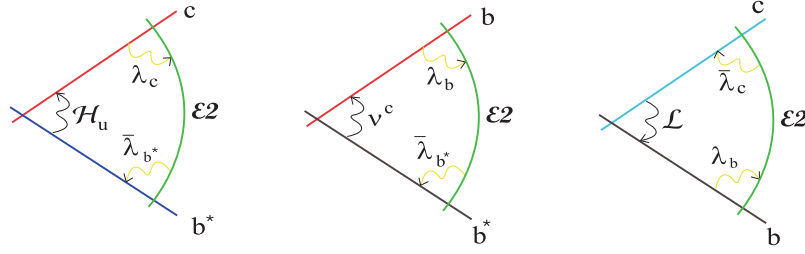


Figure 3: The zero modes on the $\mathcal{E}2$ – $D6$ intersections generating an effective Dirac neutrino mass term.

4. Neutrino masses

The experimental evidence of non-zero neutrino masses and mixing urges for a rather profound extension of the Standard Model, i.e., the introduction of the right-handed neutrino. This makes possible the implementation of the attractive see-saw mechanism in order to resolve the puzzle of the tiny neutrino masses. We have seen that in the present D-brane construction, the right-handed neutrino can arise from an open string whose one endpoint is attached on the $U(2)$ (i.e. $D6_b$) brane-stack and the other on its corresponding mirror brane-stack obtained under the orientifold action $\Omega\mathcal{R}D6_b$. Under the charge assignment of Table 1 a Dirac mass term for the neutrino is not allowed at the perturbative superpotential. The right-handed neutrino however, is an electrically neutral singlet, while the $U(1)_b$ charge does not play any role in the hypercharge generator, thus any of the two possible neutral singlets with $\mathcal{Q}_{b_{\nu^c}} = \pm 2$ may appear in the spectrum. Had we chosen to keep the neutral state with $\mathcal{Q}_{b_{\nu^c}} = +2$, a direct tree-level Dirac mass term $\lambda_{ij}^\nu L_i \nu_j^c H_u$ would be possible as was discussed in the previous section. In the alternative case of $\mathcal{Q}_{b_{\nu^c}} = -2$, an effective non-perturbative term through instanton effects can be generated from the $\mathcal{E}2$ – $D6$ intersections of figure (3) provided $I_{\mathcal{E}2b} = -2$, so that the $U(1)_b$ charge excess $\sum_j \mathcal{Q}_{b_j} = -4$ is canceled by the instanton factor of the non-perturbative coupling. Instanton effects can also generate Majorana masses for the singlet right-handed neutrino which are of the form [12],[13],[22],[23]

$$M_{N,j} = \lambda_{N,j} e^{-S_{\mathcal{E}}} M_S \quad (24)$$

where $\lambda_{N,j} = |\lambda_{N,j}| e^{X_j}$ summarizes contributions of the Kähler potential and the one-loop determinant [12].

Compared to a direct tree-level perturbative mass term, the instanton generated Dirac neutrino mass in this model is more attractive for the following two reasons. First, there is a suppression factor analogous to that of the Majorana right-handed neutrino mass. As a consequence, the see-saw mechanism $m_\nu \propto e^{-S_{\mathcal{E}}} \frac{m_D^2}{M_S}$ can be implemented with a moderate value of the string scale. A second observation is that in the derivation of the neutrino mass term though instanton effects, the corresponding disk diagrams do not involve simultaneously the matter fields \mathcal{L} and ν^c . This implies that the Dirac neutrino Yukawa coupling in the 3×3 flavor space is factorizable. To exhibit the advantage of such a simplified structure of the neutrino mass matrix, we assume as an example the case of almost diagonal charged

lepton mass matrix. We further assume symmetric types of mass matrices and parametrise the Dirac neutrino mass matrix in terms of two real parameters as follows. Let the vector

$$\vec{\lambda}^\nu = \{1, r \cos \theta, r \sin \theta\} \quad (25)$$

represent the three λ_i 's in flavor space. Then, we write the instanton induced factorizable Dirac neutrino Yukawa coupling as follows

$$y_{ij} = e^{i\phi_{ij}} \lambda_i^\nu \lambda_j^\nu \quad (26)$$

where ϕ_{ij} are possible phases. We can write the Dirac neutrino mass matrix as follows

$$m_D^\nu = \begin{pmatrix} 1 & r \cos(\theta) & r \sin(\theta) \\ r \cos(\theta) & e^{i\phi_1} r^2 \cos^2(\theta) & e^{i\phi_2} \frac{r^2}{2} \sin(2\theta) \\ r \sin(\theta) & e^{i\phi_3} \frac{r^2}{2} \sin(2\theta) & e^{i\phi_4} r^2 \sin^2(\theta) \end{pmatrix} \quad (27)$$

It is easy now to pick up choices of the parameters that reproduce the known neutrino data. Let us for the sake of simplicity concentrate on a specific example, choosing a set of phases $\chi_1 = 0$ and $\phi_i = \chi_{2,3} = \pi$. If we further assume the expansion $r = 1 + \epsilon + O(\epsilon^2)$, we get

$$m_\nu^{eff.} \equiv m_D M_N^{-1} m_D^T \approx \begin{pmatrix} -\epsilon & \cos(\theta) & \sin(\theta) \\ \cos(\theta) & -\epsilon \cos^2(\theta) & -\frac{\epsilon}{2} \sin(2\theta) \\ \sin(\theta) & -\frac{\epsilon}{2} \sin(2\theta) & -\epsilon \sin^2(\theta) \end{pmatrix} m_0 \quad (28)$$

where m_0 is of the order $e^{-S_\epsilon} m_W^2 / M_S$ and should be in the sub-eV range. From the point of view of the experimental and other observational data for neutrino masses and their mixing, this is a very interesting example of a neutrino mass matrix. This gives bimaximal mixing and an inverted hierarchy spectrum with two almost degenerate eigenmasses $m_{\nu_1} \sim (-1 - \epsilon)m_0$, $m_{\nu_2} \sim (1 - \epsilon)m_0$, and $m_{\nu_3} = 0$, which, if small corrections are received from charged lepton mixing, could fit the neutrino data[24].

5. Conclusions

In this work we have discussed in some detail vital issues regarding the Yukawa sector in an effective Standard Model Theory obtained from a minimal D-brane configuration [6] equivalent to the gauge symmetry $U(3) \times U(2) \times U(1)$. More precisely, we have explored the implications of the custodial global $U(1)$ symmetries (indispensable in these constructions) on the fermion masses using sensible, phenomenologically consistent variants of the fermion and Higgs spectra.

In the desirable and elegant case of a minimal spectrum consisting just of the MSSM chiral matter, the right-handed neutrino and the one doublet Higgs pair, several Yukawa couplings are absent in the perturbative superpotential. It was found that the suggested mechanism based on string space-time instanton contributions can generate the missing Yukawa couplings which fill in the zero entries of the perturbative quark mass matrices.

These corrections are found to be suppressed by an exponential factor of the form $e^{-S_{\mathcal{E}}}$ where $S_{\mathcal{E}}$ is the classical instanton action. This is a generic characteristic with rather exciting and unprecedented implications on the low energy phenomenology of these constructions. Thus in a realistic D-brane Standard Model analogue one could envisage to interpret the fermion mass spectrum through these instanton effects, attributing the observed mass hierarchy to the aforementioned exponential suppression of the non-perturbative couplings. Furthermore, a rather interesting Dirac neutrino mass matrix texture is predicted and heavy Majorana masses are also induced from similar non-perturbative effects. As an example, we have presented a simple parametrization of this specific texture which easily reconciles the neutrino experimental data.

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